Length-slope factors for the Revised Universal Soil Loss Equation: Simplified method of estimation

Ian D. Moore and John P. Wilson

ABSTRACT: The combined length-slope (LS) factor in the Universal Soil Loss Equation (USLE) is a measure of the sediment transport capacity of overland flow. A dimensionless sediment transport capacity index that is a non-linear function of specific discharge and slope was derived by considering the transport capacity limiting sediment flux in the Hairsine-Rose, WEPP, and catchment evolution erosion theories. For a two-dimensional hillslope, the index is equivalent to the combined LS factors in the Revised Universal Soil Loss Equation (RUSLE), but it is simpler to use and conceptually easier to understand. A major advantage of the index is that it can be easily extended to three dimensional terrain.

THE Universal Soil Loss Equation (USLE) was empirically derived from over 10,000 plot-years of data *(24)* and has recently been revised *(12, 13, 18)*. The USLE and the Revised Universal Soil Loss Equation (RUSLE) can be written as:

$$A = RKLSCP \qquad [1]$$

where A is the soil loss, R is the rainfall-runoff erosivity factor, K is a soil erodibility factor, L is a slope-length factor, S is a slope-steepness factor, C is a cover-management factor and P is a supporting factor. Land use and management are represented by CP and can, with some difficulty, be inferred by remote sensing combined with ground-truthing. Climate erosivity is represented by R and can be computed directly from a knowledge of rainfall intensities and amounts; it varies on a regional scale. Soil erodibility is represented by K, and in the United States values of K have been interpolated or measured for all mapped soil series as part of the Soils-5 database that is derived from County Soil Surveys. Soil series are mapped at scales of 1:15,000 to 1:20,000 in these surveys. The effects of topography and hydrology on soil loss are characterized by the combined LS factor. Table 1 presents expressions for the L and S factors in both USLE and RUSLE. Soil loss predictions are more sensitive to slope steepness than slope length. The combined LS factors for the USLE-LS and the RUSLE-LS are compared in Figure 1. Only values of slope-length <100m, slope-steepness <25%, and USLE-LS and RUSLE-LS <5 are included in this figure.

Estimation of the LS factor poses more problems than any of the other factors in the USLE *(18, 23)* and is a particular problem in applying it to real landscapes as part of a Geographic Information System (GIS). Some of these problems follow from a number of implicit assumptions concerned with runoff generation and sediment transport imbedded in the equation, notably that runoff is generated uniformly over a catchment, runoff occurs via the infiltration excess mechanism (i.e., Hortonian overland flow) and ignores saturation overland flow, and sediment deposition is not represented, which represents a major practical problem because the model does not differentiate those parts of landscapes experiencing net erosion and those areas experiencing net deposition (i.e., the lower ends of concave slopes). The USLE has been adapted to variable hillslopes (4, 5), but only applies to those areas experiencing net erosion. Other problems and limitations follow from the implicit division of landscapes into hillslopes. Soil loss is best estimated for points or small areas (i.e., grid cells) when USLE is applied to large areas rather than fields and hillslopes *(5)*. This one-dimensional structure means that the equation cannot handle converging and diverging terrain (i.e., real 3-D landscapes).

A simplified method of estimating the LS factors in RUSLE is presented that can be easily extended to estimating soil loss in complex 3-D terrain. It may also help distinguish areas experiencing net erosion and those experiencing net deposition. The method is derived by considering the steady-state, sediment transport limiting case predicted by three erosion theories.

Erosion theory

Three approaches that represent the "state-of-the-art" in erosion modeling are described here. They include sophisticated physically-based or process-oriented models, as well as both dynamic and steady-state models and cover simulation time scales ranging from seconds to thousands of years (geological time scales).

Water Erosion Prediction Project (WEPP) theory. The United States Department of Agriculture (USDA) is developing improved, process-based erosion-prediction models aimed at replacing the USLE by 1995. WEPP is to be delivered in three versions: profile, watershed, and grid, with the profile version being the direct replacement of the USLE *(10)*. WEPP is based on Foster and associates *(2)* concept that divides erosion into an interrill component representing detachment and transport by raindrops and very shallow flows and a rill component that represents net erosion or deposition in rills. Rill detachment is modeled as a function of excess hydraulic shear *(9)*. WEPP per- forms its internal calculations on a per rill area basis.

The theory is encapsulated by the following steady-state sediment continuity equation for a rill:

$$dq_s / ds = D_f + D_i \qquad [2]$$

where q_s is the sediment flux ($kgm^{-1}s^{-1}$), D_i is the interill sediment delivery rate to the rill ($kgm^{-2}s^{1}$) and D_f is the net erosion or deposition rate in the rill ($kgm^{-2}s^{-1}$) *(1, 3)*. The shallow flow hydraulics in the interrill areas are not directly modeled, but their effects on delivering sediment to the rill are lumped with the rainfall kinetic energy term and modified by a land slope adjustment factor in the expression for D_i,

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which can be written as:

$$D_i = K_i I_e^2 C_g C_c S_f (R_s/w) \quad [3]$$

where K_i is the interrill erodibility ($kgsm^{-4}$), I_e is the effective rainfall ($m\ s^{-1}$) C_g is a ground cover adjustment factor, C_c is a canopy cover adjustment factor, S_f is a slope adjustment factor ($= 1.05 - 0.85\ e^{4 \sin a}$), a is the slope of the land surface towards the rill, R_s is the spacing between rills (m per rill) and w is the rill width (m) *(2,11)*. The net erosion or deposition rate in the rill, D_f is:

$$D_f = \phi\ (T_c - q_s) \quad [4]$$

where T_c is the sediment transport capacity in the rill ($kgm^{-1}s^{-1}$) $\phi = \beta v_s/q$ for net deposition in the rills (when $T_c < q_s$) and $\phi = D_c/T_c$ for net soil detachment in the rills (when $T_c > q_s$), q is the water flux ($m^3 m^{-1} s^{-1}$), v_s is the sediment settling velocity (ms^{-1}) and D_c is the detachment capacity of rill flow ($kgm^{-2}s^{-1}$). For rainfall conditions $\beta = 0.5$ and for non-rainfall or snowmelt conditions $\beta = 1.0$. D_f is positive when there is net erosion and negative when there is net deposition. The detachment capacity of rill flow can be expressed as:

$$D_c = K_r \tau\ (1 - \tau_o/\tau) \text{ for } \tau > \tau. \quad [5]$$
$$\text{and } D_c = O \text{ for } \tau < \tau_o$$

where τ is the flow shear stress acting on the soil particles (Pa), τ_o is a threshold shear stress (Pa) and K_r is a rill erodibility parameter (sm^{-1}). The sediment transport capacity is represented by an approximation of the Yalin sediment transport equation:

$$Tc = k_t \tau 3/2 \quad [6]$$

where k_τ is a transport coefficient ($m^{1/2} s^2 kg^{-1/2}$).

Hairsine-Rose theory. The Hairsine-Rose theory is a process-based approach that recognizes raindrop impact and surface flow as the agents causing erosion of surface soils. Rainfall detachment, entrainment (detachment by overland flow), rainfall re-detachment and re-entrainment of deposited sediment and deposition are modeled as separate processes *(6, 7, 8)*. The theory is an outgrowth of concepts originally developed by Rose *(20, 21)*. The theory is encapsulated by the following one-dimensional sediment continuity equation:

$$\partial q_{si} + \partial (C_i h) \partial s \partial L = \quad [7]$$
$$r_i + r_{di} + e_i + e_{di} + r_{gi} - d_i$$

where q_{si} ($= qC_i$) is the sediment flux ($kgm^{-1}s^{-1}$) in the direction of flow (s), q is the water flux (specific discharge), C_i is the sediment concentration (kgm^{-3}), h is the depth of overland flow (m), r_i, r_{di} e_i, e_{di}, and d_i are the rainfall detachment, rainfall re-detachment, entrainment, re-entrainment and deposition, respectively ($kg\ m^{-2}s^{-1}$) and subscript i refers to each of N sediment settling velocity classes (i) with an equal mass of soil in each class. The gravity process rate is r_{gi} and represents contributions from headcut collapses and slumping of rill walls *(7)*.

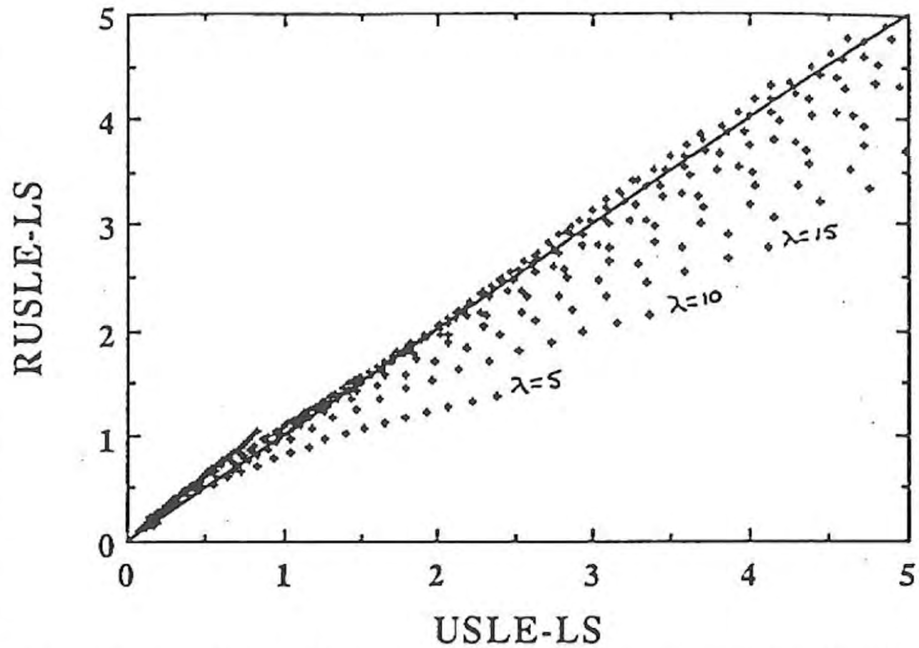

Figure 1. Comparison of the USLE-LS and RUSLE-LS factors, where λ is the slope length (m).

Table 1. LS factors in the Universal Soil Loss Equation (USLE) and Revised Universal Soil Loss Equation (RUSLE) (λ=slope length in meters; β=slope angle in degrees).

	LS Factors			
	S		$L = (\lambda/22.13)^m$	
USLE (22)	$65.4 \sin^2\beta + 4.56 \sin\beta + 0.0654$		m=0.5 m=0.4 m=0.3 m=0.2	$\tan\beta > 0.05$ $0.03 < \tan\beta \le 0.05$ $0.01 < \tan\beta \le 0.03$ $\tan\beta \le 0.01$
RUSLE (12,13)	$10.8 \sin\beta + 0.03$ $16.8 \sin\beta - 0.50$ $3 \sin^{0.8}\beta + 0.56$ $(\sin\beta/0.0896)^{0.6}$	$\tan\beta < 0.09$ $\tan\beta \ge 0.09$ $\lambda \le 4m$ thawing soils* with $\tan\beta \ge 0.09$	$m = F/(1+F)$ wher $F = (\sin\beta/0.0896)/(3 \sin^{0.8}\beta + 0.56)$* or F=0 when there is deposition when λ=4m λ≤4m equal to 4m.	
Moore & BURCH[1] (14,15)	$LS = A_s/22.13)^m (\sin\beta/0.0896)^n$ where m=0.4,n=1.3, and A_s=specific catchment area			

* Assumes a moderate rill/interrill ratio (13).

[1] Derived from unit stream power theory.

Rainfall detachment, entrainment and deposition can be expressed as follows:

Rainfall Detachment:

$$r_i = (1-H)\ k\ C_e\ i^p/N \quad [8a]$$

$$r_{di} = H\ k_d C_e i^p\ f_{di} \quad [8b]$$

Entrainment:

$$e_i = (1-H)\ \eta/NE\ (\omega - \omega_o) \quad [9a]$$
$$\text{for } \omega > \omega_o$$

$$edi = H \eta \alpha_i f_{di}/g(A/fs - f)(\omega - \omega_o/h) \quad [9b]$$
$$\text{for } \omega > \omega_o$$

Deposition:

$$d_i = \alpha_i v_{si} C_i \quad [10]$$

where H is the fraction of the soil shielded by a deposited layer, k and k_d are measures of the detachability (kgs m-4) of the original and deposited soil, respectively, C_e is the fraction of soil surface exposed to raindrop impact, i is the rainfall rate ($m\ s^{-1}$), p is a nondimensional exponent, f_{di} is the fraction of particles (on a mass basis) is settling velocity class i in the deposited layer, η is the fraction of the available stream power for entrainment, E is the energy required to entrain a unit mass of soil or specific energy of entrainment (Jkg^{-1}) ω is the stream power (watts m^{-2}), ω_o is the threshold stream power, α_i is the ratio of the sediment concentration next to the bed to the mean concentration across the entire depth (C_i) so that $\alpha i \geq 1$, ρ is the density of the sediment laden water (kgm^3) (ρ=1.000 + 0.615C), ρ_s is the density of detached soil or soil aggregates (kgm^3), h is the depth of flow (m), and v_s is the sediment settling velocity (ms^{-1}). The depositabilty of the sediment is equal to $\Sigma v_{si}/N$.

Equations 8 and 9 assume that rainfall detachment and entrainment are nonselective whereas equation 10 shows that deposition is highly selective. The re-entrainment process represented by equation 9b assumes that the deposited soil has no cohesive strength. The stream power used in equations 9a and 9b is the stream power per unit wetted area *(19)*, $\omega = \rho g q \sin\beta$, where ρg is the unit weight of water and β is the slope of the energy grade line, which is assumed equal to the land slope. An equivalent expression for stream power is $\omega = \tau v$, where τ is the shear stress and v is the flow velocity. In equation 9b the ω/h term is equivalent to $\rho g v \sin\beta$, where $v\sin\beta$ is the unit stream power (watt N^{-1} or ms^{-1}), defined as the stream power per unit weight of water. In both equations 9a and 9b e_i=0 and e_{di}=0 when $\omega \leq \omega_o$. The fraction of available stream power for entrainment (η) is about 0.1, although it increases to 0.2 for low stream powers, and typical values of E in equation 9a are 20-30 Jkg^{-1} for cultivated soils and 100-150 Jkg^{-1} for rangeland soils (Hairsine, personal communication). Also, the exponent p in equation 8a is usually assumed to to be -1. The soil rainfall detachability terms in equations 8a and 8b can be written as functions of the maximum detachability (k_o and k_{do}) and water depth (h):

$$k = k_o(h_o/h)^b \text{ for } h>h_o \qquad [11a]$$

and $k = k_o$ for $h \leq h_o$

$$k_d = k_{do}(h_o/h)^b \text{ for } h>h_o \qquad [11b]$$

and $k_d = k_{do}$ $h \leq h_o$

where h_o is a threshold water depth.

Catchment evolution theory. Willgoose and associates *(22)* have recently proposed a hillslope and catchment evolution model that explicitly differentiates between the sediment transport behavior in channels and on hillslopes via coupled flow and sediment continuity equations in the hillslope and channel. Both diffusive (function of slope only, e.g., raindrop splash, soil creep and rock slide) and fluvial (function of slope and discharge) sediment transport processes and tectonic uplift are represented. Channel initiation is modeled as a threshold process that is nonlinearly related to slope and discharge. The governing sediment continuity equation and channel indicator function can be expressed, respectively, as: [12a]

$$\partial z = c_o(x,y,) + 1/\rho_s(1-n)\ (\partial_q + \partial_q / \partial x\ \partial y) + D_z\ (\partial^2 z + \partial^2 z / \partial x^2\ \partial y^2)$$

[12b]

$$\partial Y/\partial t = d_t\ [0.0025a/a^t - 0.1Y + Y^2/ 1+9Y2]$$

where z is elevation, c_o is the rate of tectonic uplift, ρ_s is the density of eroded material, n is the porosity of material before erosion and after deposition, D_z is a diffusivity constant, Y is a channel indicator function (=0 hillslope; =1 channel; $0 \leq Y \leq 1$), d_t is a rate constant for channel growth, a is a channel initiation function [$= \phi_1 q^{m1}(\sin\beta)^{n1}$], a_t is a channel initiation threshold, β is the slope angle in the direction of steepest descent, and ϕ_1, m1, and n1 are constants. The sediment flux, q_s, is a function of the water flux, q, and the land surface slope β:

$$q_s = \phi_2 q^m\ (\sin\beta)^n \qquad [13]$$

where m and n are constants and ϕ_2 is a rate constant for sediment transport, that is different for hillslopes and channels. In equation 12b Y=0 for $a<a_t$, goes into a transition when $a=a_t$, increasing to Y=1 at a speed dependent on the channel growth rate constant, d_t, and once it reaches a value of 1 remains there. Most of the hillslope evolution models developed in the last 20 years solve a sediment continuity equation similar to equation 12a.

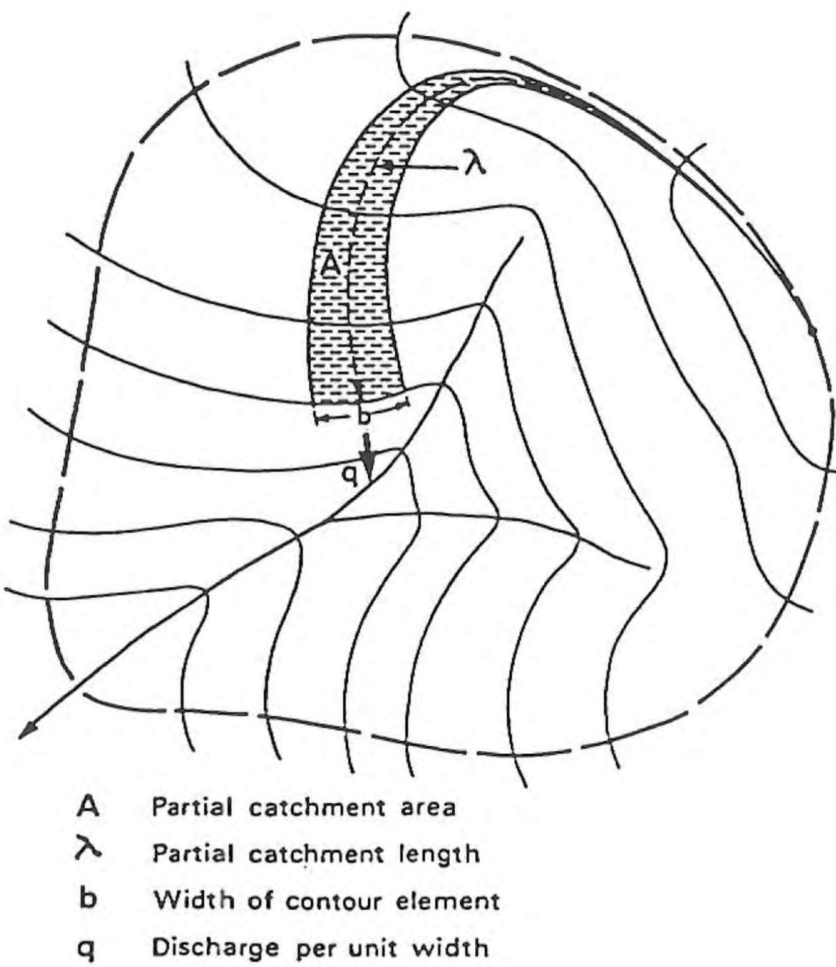

Figure 2. Schematic representation of specific catchment area, A_s = A/b [adapted from (15)].

General sediment transport equations. Using dimensional analysis, Julien and Simons *(9)* have shown that most sediment transport equations can be expressed in the following general form:

$$q_s = \phi_2\, q^m (\sin\beta)^n i^\delta (1-\tau_0/\tau)^\varepsilon \qquad [14]$$

where i is the rainfall intensity, ϕ_2, n, m, δ and ε are experimental or physically-based coefficients, and the other terms are as previously defined. The first three terms ($\sin\beta$, q, i) represent the potential erosion or transport by flow, which is reduced by the last term (the shear stress term) reflecting the soil resistance to erosion *(9)*. When τ_0 is small compared to τ, the shear stress term can be neglected. The rainfall intensity term is also ignored in many sediment transport equations (i.e., δ = 0), but this is only strictly true for turbulent flows in deep channels.

If we assume rainfall excess is generated uniformly over a catchment then $q = A_s i_e$, where A_s is the upslope contributing area per unit width of contour (or rill) or the specific catchment area ($m^2\ m^{-1}$) and i_e is the rainfall excess rate ($m\ s^{-1}$). A schematic representation of the specific catchment area is presented in Figure 2. For a 2-D hillslope where there is no flow convergence or divergence $A_s = \lambda$, the slope length.

Transport limiting case

For large runoff and erosion events the "transport limiting" case, where the sediment flux is limited only by the ability of the flow to carry the sediment, is likely to be the dominant influence on the pattern of erosion in landscapes. With the WEPP theory this transport limiting case occurs when $q_s = T_c$. We can represent the overland flow hydraulics as uniform turbulent flow using Manning's equation. The WEPP theory assumes that sediment is transported from a site by concentrated flow in rills. By approximating the relationship between hydraulic radius, R, and rill cross sectional area, A, by $R = UA^{1/2}$, where U is a rill shape factor *(16)*, equation 6 can be written in terms of the specificlc discharge, q (discharge per unit width of catchment, not the discharge per unit width of rill), and slope angle, β: [15a]

$$T_c = k_t(\rho g)^{1.5} (R_s n U^{-2})^{0.56} q^{0.56} (\sin\beta)^{1.22} \text{ and with } q = i_e A_s$$

[15b]

$$T_c = k_t(\rho g)^{1.5}(R_s i_e n U^{-2})^{0.56} A_s^{0.56} (\sin\beta)^{1.22}$$

where n is Manning's roughness coefficient and R_s is the rill spacing (m per rill). If we write equation 15 in a dimensionless form so that T_c^*, the dimensionless sediment transport capacity, is unity when $A_s = 22.13\ m^2\ m^{-1}$ and $\tan\beta = 0.09$ (as with the LS factor in the USLE), then: [16]

$$T_c^* = (A_s/22.13)^{0.56} (\sin\beta/0.0896)^{1.22}$$

where the exponents 0.56 and 1.22 are equivalent to the slope-length and slope-angle exponents, m and n, respectively, in the LS factor in the USLE. If shallow sheet flow were assumed rather than concentrated flow in rills then the exponents in equation 16 would be 0.9 and 1.05, respectively.

In the Hairsine-Rose theory the equivalent "transport limiting" case under the steady-state sediment flux occurs when $\partial(C_i h)/\partial t = 0$ and H=1 in equations 7 to 10 *(7)*, which corresponds to the condition where there is a layer of deposited sediment over the

Figure 3. Comparison of different forms of the dimensionless sediment transport capacity index versus the length-slope factors in the Universal Soil Loss Equation (USLE) and Revised Universal Soil Loss Equation (RUSLE).

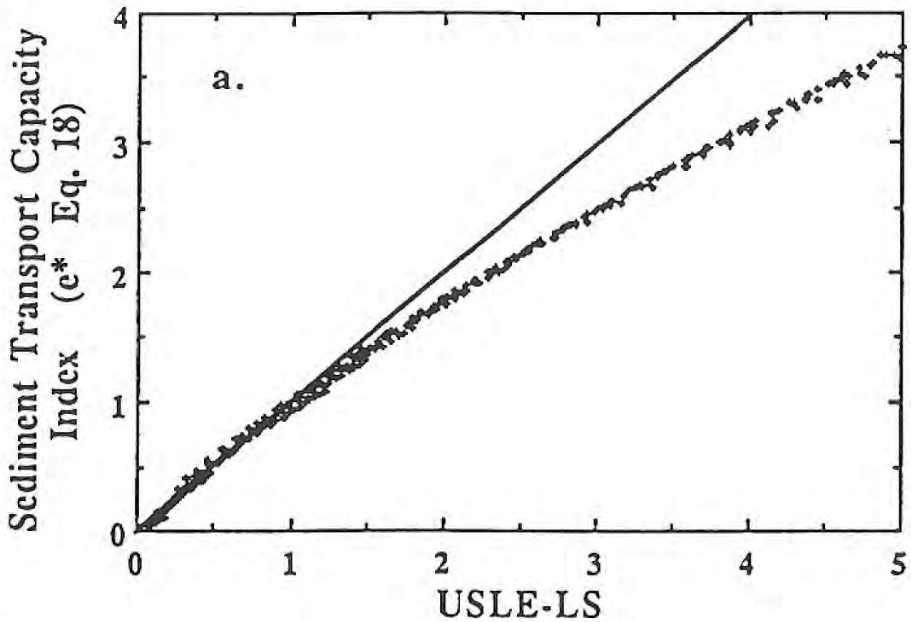

Figure 3a. Hairline-Rose theory-based index versus USLE-LS.

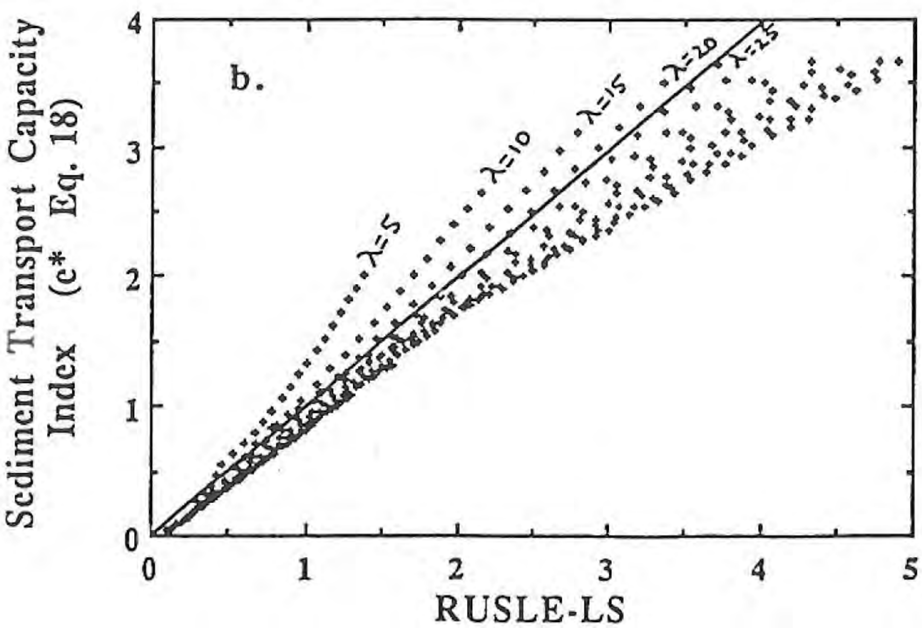

Figure 3b. Hairsine-Rose theory-based index versus RUSLE-LS (λ = slope length in m).

entire hillslope. If the threshold term, ω_o/h in equation 9b is small compared to ω/h, and can therefore be neglected, then the re-entrainment rate given by equation 9b can be rewritten as: [17]

$$e_{di} = H\eta\, \rho\alpha_i f_{di}/_n 0.6\, (\rho_s/\rho-\rho) q^{0.4} (\sin\beta)^{1.3} = H\eta\, \rho\alpha_1 f_{di}/_n 0.6\, (\rho_s/\rho_s-\rho)\, i_e^{0.4} A_s^{0.4} (\sin\beta)^{1.3}$$

Again, writing equation 17 in dimensionless form like equation 16, a dimensionless re-entrainment rate, e^*, can be derived: [18]

$$e^* = (A_s/22.13)^{0.4} (\sin\beta/0.0896)^{1.3}$$

which is the unit stream power based length-slope factor proposed by Moore and Burch *(15)* (Table 1).

Equation 13, used in the catchment evolution model, can also be reduced to a dimensionless form having the same structure as both equations 16 and 18. The S factor in the RUSLE for thawing soils is also expressed in this form, but with the slope exponent n=0.6 (Table 1).

Results and discussion

The dimensionless re-entrainment rate derived from the Hairsine-Rose theory and derived independently by Moore and Burch *(14, 15)*, e^*, is compared to the LS factors in the USLE (USLE-LS) and RUSLE (RUSLE-LS) (Table 1) in Figures 3a and 3b, respectively, for the case where $A_s = \lambda$, and λ is the slope-length. There is a strong monotonic function relating USLE-LS to e^*, with e^* < USLE-LS for USLE-LS values >1.5, which is consistent with the widely held view that the USLE overpredicts LS values at higher slopes and longer slope-lengths. Figure 3b shows that there is considerable scatter in the RUSLE-LS versus e^* relationship. However, for $A_s=\lambda=22.13\ m^2\ m^{-1}$ (i.e., $A_s/22.13=1$) there is very close agreement, indicating that the theoretical exponent of 1.3 on the slope term of e^*, is quite accurate. Only values of slope-length < 100 m, slope-steepness < 25% and USLE-LS and RUSLE-LS<5 are included in Figure 3.

Figure 3c shows that there is also good agreement between the dimensionless sediment transport capacity, T_c^*, that is derived from the WEPP theory and RUSLE-LS. To a large degree this is expected as the LS factors developed for the RUSLE by McCool et al. *(12, 13)* were derived in part by applying the Foster and Meyer *(1)* theory, which is the basis of the WEPP model.

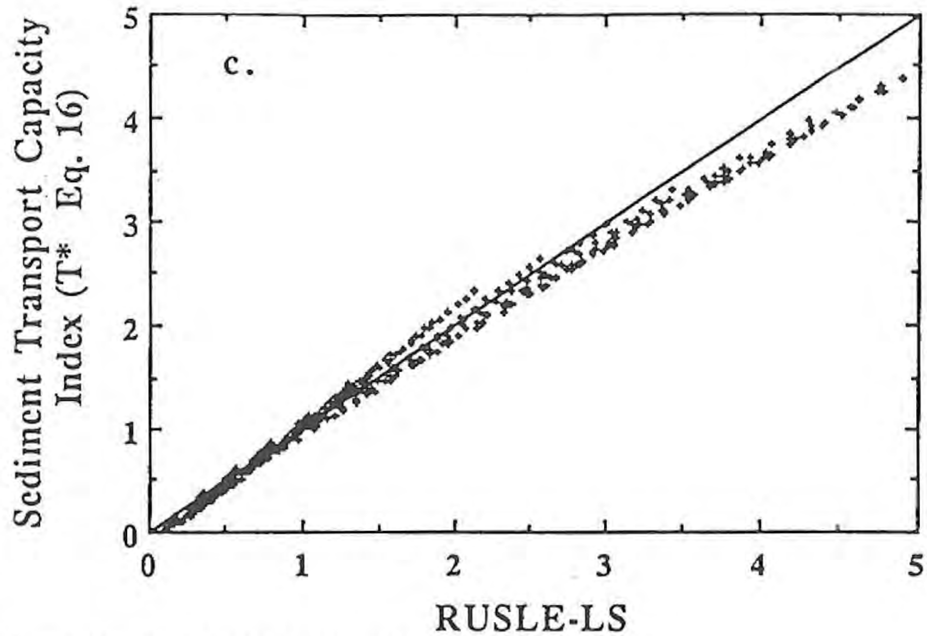

Figure 3c. WEPP theory-based index versus RUSLE-LS.

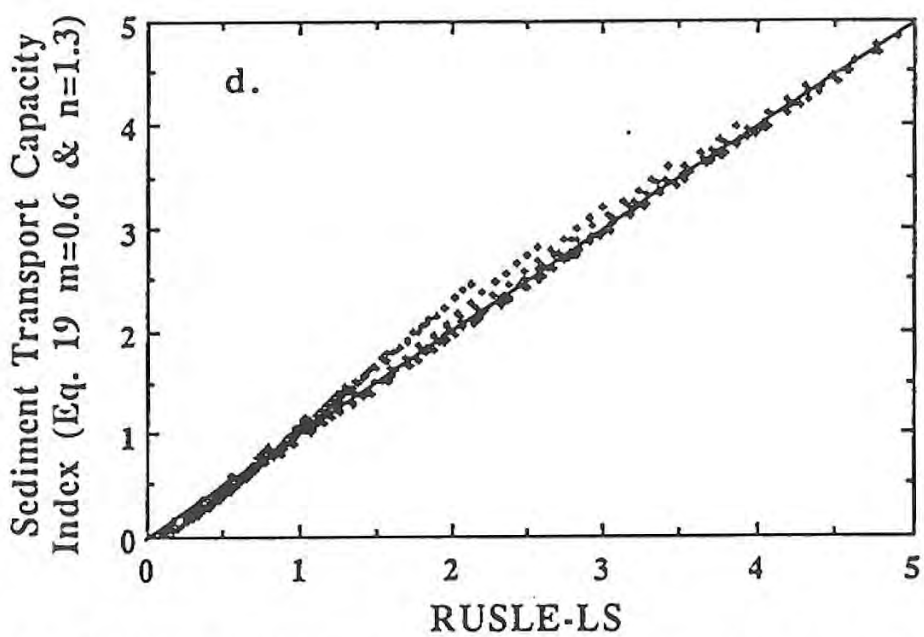

Figure 3d. Generalized dimensionless sediment transport equation with m=0.6 and n=1.3 (Eq.) versus RUSLE-LS.

However, equation 18 is functionally simpler and easier to use than the RUSLE-LS factors.

The best fit between RUSLE-LS and an equation of the form of equations 14 and 18 occurs when the area and slope exponents (m and n) are 0.6 and 1.3, respectively (Figure 3d). These results suggest that the combined LS factor in the USLE and RUSLE are measures of the sediment transport capacity of the flow. Furthermore, they show that a sediment transport equation of the form of equation 14 or written in dimensionless form as: [19]

$$T_c^* = (A_s/22.13)^m (\sin\beta/0.0896)^n = LS$$

with m=0.6 (0.4 to 0.6) and n=1.3 (1.2 to 1.3) can be used to map the effects of hydrology, and hence 3-D terrain, on soil erosion in natural landscapes. The A_s term can characterize the effect of converging and diverging terrain on soil erosion, unlike the λ term in the USLE and RUSLE, which is only applicable to 2-D, non-converging and non-diverging hillslopes. For predicting erosion at a point, equation 19 should by multiplied by (m+1), as proposed by Griffin and associates *(5)*.

Comment on
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George R. Foster

Based on the paper's title and implications in the paper, a reader might assume that the *LS* relationships developed by Moore and Wilson *(6)* are interchangable with or equivalent to the slope length-slope steepness factor *(LS)* relationships used in the Revised Universal Soil Loss Equation (RUSLE) and that the Moore and Wilson *LS* equation can be used instead of the current RUSLE *LS* relationships. Such is not the case.

Moore and Wilson concluded that the RUSLE *LS* factor is a measure of sediment transport capacity, but this conclusion is exactly the opposite of that determined by the developers of RUSLE. McCool et al. *(5)* assumed detachment-limiting conditions to derive the slope length relationships for RUSLE. This assumption produces values for the slope length exponent in the RUSLE *L* factor that vary from 0 to 1, which is the range observed in experimental data *(5)*. Furthermore, the slope length exponent in experimental data is near zero for low slopes and near 1 for steep slopes. The slope length exponent in experimental data also increases as rill erosion increases relative to interrill erosion, and the slope length exponent tends to be near 1 when rill erosion is large relative to interrill erosion. The slope length exponent relationships used in RUSLE are based on detachment-limiting theory, which very nicely describes the trends observed in experimental data.

In contrast, the transport-limiting theory assumed by Moore and Wilson gives a single value for the slope length exponent that does not vary with slope steepness, slope length, or rill erosion. That is, the transport-limiting theory does not provide a variable slope length exponent to match experimental data. Furthermore, Foster and Wischmeier *(3)* and Foster and Meyer *(2)* calculated sediment transport capacity using the Yalin equation and found transport capacity to be larger than measured sediment load for conditions represented by RUSLE. Alonso, Neibling, and Foster *(1)* showed that the Yalin equation is a good estimator of transport capacity for conditions represented by RUSLE. Based on these and other considerations, the developers of RUSLE concluded that RUSLE and its *LS* factor represent detachment-limiting conditions, not transport-limiting conditions as claimed by Moore and Wilson. Notably, Moore and Wilson made no comparisons with experimental data to justify their claim.

In their equation (19), Moore and Wilson *(6)* equated an expression for a dimensionless sediment transport capacity with the RUSLE *LS*. Regardless of whether *LS* represents detachment or transport-limiting conditions, Moore and Wilson's equation (19) is

$$T^*_c = (A_s/22.13)^m (\sin\beta/0.0896)^n = LS \quad (19)$$

where T^*_c = dimensionless transport capacity, A_s = upslope area/unit contour width, 22.13 = slope length of the RUSLE unit plot in meters, m = slope length exponent, β = slope angle, 0.0896 = sine of the 9% slope of the RUSLE unit plot, and n = slope steepness exponent. The dimensionless transport ca-

Table 1. Computation of transport capacity (sediment load) and change in transport capacity (sediment load) by segment on a 9% uniform slope using Moore and Wilson's equation 20* and m = 0.6 and n = 1.3.

Distance downslope	Transport capacity	Change in transport capacity on a segment
m	(mass/width)	(mass/width)
0	0.00	
10	0.17	0.17
20	0.26	0.09
30	0.33	0.07
40	0.40	0.06
50	0.45	0.06

*Equation 20 has been rearranged as equation 26 with ϕ = 1

pacity T^*_c is given by

$$T^*_c = T_c / T_{cu} \qquad (21)$$

where T_c = sediment transport capacity (mass/width × time) and T_{cu} = transport capacity for the RUSLE unit plot conditions.

RUSLE computes soil loss A (mass/area · time), which is the average erosion rate for the slope length λ. The equation for A is

$$A = \kappa(\lambda/22.13)^m S \qquad (22)$$

where κ = the product of the RUSLE factors RKCP (erosivity, erodibility, cover management, and supporting practices factors, respectively) and S = the RUSLE slope steepness factor. The term $\lambda/(22.13)^m$ in equation 22 is the L factor in RUSLE. The variable A differs from T_c in that A is the average soil loss from the slope length λ and has units of mass/(area × time), whereas T_c is the amount of sediment passing through a plane at the end of λ and has the units of mass/(width × time). The relationship between A and T_c is $T_c = A\lambda$, when A is considered to be a measure of transport capacity. Therefore, when A is a measure of transport capacity, multiplication of equation (22) by slope length λ gives

$$\begin{aligned} T_c &= \kappa(\lambda/22.13)^m \lambda S \\ &= \kappa\lambda^{m+1} S/22.13^m \end{aligned} \qquad (23)$$

The transport capacity for the unit slope is given by

$$\begin{aligned} T_{cu} &= \kappa(22.13/22.13)^m\, 22.13 \\ &= 22.13\ \kappa \end{aligned} \qquad (24)$$

where the slope length of the unit plot has been substituted for λ in equation (23) and S = 1 for a 9% slope. Division of equation (23) by equation (24) gives the equation for dimensionless transport capacity T^*_c, which is

$$T^*_c = (\lambda/22.13)^{m+1}\ S \qquad (25)$$

Moore and Wilson's equation (19) was developed for use in terrain modeling of three dimensional landscapes where flow converges and diverges. However, for slopes of uniform width represented by RUSLE, the A_s term in Moore and Wilson's equation (*19*) can be replaced with slope length λ.

A comparison of equation 25 with Moore and Wilson's equation 19 shows that the slope length exponent for equation (19) should be $m + 1$ and not the exponent m that Moore and Wilson showed in their equation 19.

Thus, Moore and Wilson's equation (19) is incorrect. The error in Moore and Wilson's equation 19 is easily illustrated by solving their equation 20, which is

$$\Delta T_c = \phi\,[\lambda_j^m(\sin\beta_j)^n - \lambda_{j-1}^m\ (\sin\beta_{j-1})^n] \qquad (26)$$

where λ has been substituted for A_s, the order of the terms has been reversed so that erosion rates have positive rather than negative signs, and the subscript j indicates the lower edge of a slope segment. Moore and Wilson stated that their equation (20) provides a possible measure of erosion or deposition on a slope segment or cell.

Values from the solution of their equation (20) are shown in Table 1, illustrating that Moore and Wilson's equation (20) computes an erosion rate that decreases with distance downslope, which is incorrect. The correct slope length exponent in equation 20 should be m + 1, as equation (23) shows. Moore and Wilson commented, "However, further testing is required to determine whether or not this relationship applies across most landscapes." The simple computations performed to develop Table 1 show that Moore and Wilson's equation does not apply to any landscape because of their error in the exponent on their A_s term.

For these reasons, Moore and Wilson's statement concerning the A_s (slope length) exponent in their equation (16) is also incorrect when they stated that the slope length exponent from their transport capacity equation is the same as the RUSLE L slope length exponent. As the above equations show, a value of 1 should be subtracted from their A_s exponent in equation (16), a transport capacity equation, to produce the correct slope length exponent that is comparable to the slope length exponent in RUSLE, an average erosion rate equation. Had they made this necessary subtraction, they would have obtained a slope length exponent of –0.44. Differentiation of Moore and Wilson's equation for transport capacity with respect to distance also shows this error. In this operation, the analytical form of the equation for erosion rate is $D = dT_c/dx$, where x = distance downslope, which is actually Moore and Wilson's equation (20). Applying this operation to Moore and Wilson's equation (15b) gives a −0.44 slope length exponent indicating that Moore and Wilson's transport equation computes an erosion rate that decreases with distance downslope when slope steepness is uniform, obviously an incorrect result.

Moore and Wilson concluded that the RUSLE *LS* factor could be simplified to the equation

$$LS = (\lambda/22.13)^{0.6}\ (\sin\beta/0.0896)^{1.3} \qquad (27)$$

While Moore and Wilson may consider this relationship to be sufficiently close to the RUSLE *LS* relationship for their application in terrain modeling, equation (27) is not sufficiently close to the RUSLE *LS* relationships to be used in typical RUSLE applications such as conservation planning for which RUSLE was developed. For example, the RUSLE *L* value for a 250 m, 0.5 percent slope is 1.2 while the value for *L* from equation (27) is 4.3, a difference far too great to be acceptable. Similar large differences exist for the S factor for low slopes. When McCool et al. (*4, 5*) developed the *LS* relationships for RUSLE, we were well aware of simple power relationships like equation (27), because they had been used in erosion prediction since the 1940s (*7*). When compared against experimental data (*4, 5*) the *LS* relationships used in RUSLE are clearly much superior to equation (27) considered by Moore and Wilson to be equivalent to the RUSLE *LS* relationships.

Moore and Wilson (*6*) misrepresented the deviation of the RUSLE *LS* relationships. Using only a portion of the RUSLE *LS* relationships, they fitted equation 27 to values computed with the RUSLE *LS* equations. On the basis of their results, they stated, "The best fit between RUSLE *LS* and an equation of the form of equations 14 and 18 occurs when the area and slope steepness exponents (*m* and *n*) are 0.6 and 1.3, respectively (Figure 3d). These results suggest that the combined *LS* factor in the USLE and RUSLE are measures of the sediment transport capacity of the flow." Equations 14 and 18 are sediment transport equations that have the form of equation (27). Earlier in their paper, Moore and Wilson stated, "Figure 3c shows that there is also good agreement between the dimensionless sediment transport capacity, T^*_c, that is derived from the WEPP theory, and the RUSLE *LS*. To a large degree this is expected as the *LS* factors developed for the RUSLE by McCool et al. (*4, 5*) were derived in part by applying the Foster and Meyer (*2*) theory, which is the basis of the WEPP model."

These statements by Moore and Wilson are erroneous and misrepresent how the RUSLE *LS* equations were derived. First, the RUSLE *S* factor was derived entirely from empirical data with no consideration of transport capacity or any erosion model. Second, the *L* factor in RUSLE was derived using erosion theory with separate terms for rill and interrill erosion. A totally detachment-limiting assumption with absolutely no connection to a sediment transport equation was used to derive the equations for the RUSLE *L* factor. Since the *L* equation in RUSLE was derived entirely from detachment-limiting theory, Moore and Wilson's claim that the fit of their equation to RUSLE *LS* values shows that the RUSLE *LS* is a measure of transport capacity is erroneous and completely misrepresents how the RUSLE *L* factor was derived and what it represents.

To conclude, the equations derived by Moore and Wilson for *LS* are not equivalent to the *LS* relationships in RUSLE. RUSLE represents detachment-limiting and not transport-limiting conditions. Moore and Wilson's LS equation should not be used in RUSLE because results will differ greatly from estimates that RUSLE computes using equations recommended by its developers. The *LS* equations recommended for RUSLE were developed from an extensive analysis of experimental data and rigorous theory for erosion mechanics consistent with the erosion processes represented by RUSLE.

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Reply to comments by Foster

Ian D. Moore and John P. Wilson

In the past, we have been able to model specific point and two-dimension (2-D; i.e., hillslope) erosional processes in considerable detail. There have been few studies that have modeled the spatial variability of erosion processes in real 3-D terrain. Our research aims to develop approximate methods that can provide a knowledge-based approach to environmental analysis and can be embedded within the data analysis subsystems common to most geographic information systems (GIS). After examining several process-oriented erosion models, such as WEPP and the Hairsine-Rose models, we concluded that RUSLE and generalized flux equations represented a reasonable compromise between physical realism and degree of sophistication for examining the spatial patterns of erosion in complex landscapes.

Equation (19), Moore and Wilson (1992)

Both WEPP and RUSLE were derived from the same conceptual theory: that erosion processes can be divided into rill and interrill components. The theory ignores the effects of re-entrainment and redetachment of previously deposited sediment, which has quite different physical properties to those of the parent soil. In WEPP, rill detachment is a function of the difference between the sediment flux and the sediment transport capacity, whereas in RUSLE the rill soil loss per unit area is a function of excess boundary shear stress. As will be demonstrated later, the form of this relationship means that rill soil loss in RUSLE is a linear function of sediment transport capacity. Hence, although the transport limiting case is not considered in RUSLE, the form of the equations ensures that the rill soil loss is implicitly a measure of the sediment transport capacity and vice versa. Thus, although USLE and RUSLE do not represent transport-limiting conditions per se, the LS factor is a measure of the dimensionless sediment transport capacity given by Moore and Wilson's *(10)* equation (16) for the rill-erosion dominated condition.

In Moore and Wilson *(10)*, it was stated that the transport-limiting case occurs when $q_s \geq T_c$ (not $q_s = T_c$; publisher's typographical error, which Foster was aware of, based on personal communication with the authors and the publisher). Equation (15b) in Moore and Wilson *(10)* is the correct formulation of the sediment transport capacity, based on the simplified Yalin equation used in WEPP [equation (6) in Moore and Wilson *(10)*]. From that, we derived a dimensionless sediment transport equation T^*_c [equation (16)], not a dimensionless soil loss equation. This equation is mathematically and functionally correct. Foster appears to have assumed that Moore and Wilson *(10)* incorrectly said that $q_s = T_c$, and then incorrectly said that the sediment load $Y = q_s = T_c$, when in fact, as Foster rightly points out, it should be $Y = q_s/A_s$, where A_s is the specific catchment area (m^2m^{-1}), and $A_s = \lambda$, for an idealized, non-converging, and non-diverging hill slope. This assumption was not made by Moore and Wilson *(10)*.

Moore and Wilson *(10)* then went on to show empirically that the form of the equation for T^*_c is functionally similar to the *LS* factors in RUSLE. We will show later that in fact this is only strictly true for the rill-limiting case. Because of this, we do agree with Foster that the slope-length exponent is not a constant. However, we will show that the range of values should be 0 to 0.56, rather than 0 to 1, as proposed by Foster and McCool et al. *(6)*, which is incorrect, based on the RUSLE slope length theory.

Foster's arguments, built around his equations (21) to (25), are irrelevant to our paper. To show why, we must go back to the theoretical derivation of the *LS* relationship for RUSLE by McCool et al. *(6)*. In RUSLE, the *L* factor is based on the relative magnitude of rill and interrill erosion for detachment limiting conditions. Rill soil loss per unit area, D_f, is expressed as

$$D_f = K_r \tau^{3/2} C_r \tag{28}$$

where τ is the boundary shear stress, K_r is the rill soil erodibility factor, and C_r is the rill soil management factor. The rill sediment transport capacity is also a function of the *3/2* power of shear stress *(2)*, so that rill soil loss ($kg\ m^{-2}s^{-1}$) can be written in terms of the rill sediment transport capacity, $T_c(K_g m^{-1} s^{-1})$, as

$$D_f = (K_r C_r / K_t) T_c \tag{29}$$

where K_t is the rill sediment transport coefficient. Hence, rill soil detachment is a linear function of rill sediment transport capacity. Using the relationship for rill sediment transport capacity given by equation (15b) in Moore and Wilson *(10)*, the rill soil loss per unit area can be written as

$$D_f = g_1\, A_s^{0.56} (\sin\beta)^{1.22} \tag{30}$$

where g_1 is a constant. This is the rill soil loss per unit at a point. The average soil loss per unit area for a hillslope or catchment Y can be obtained by integrating equation (30) with respect to the specific catchment area A_s, and then dividing by A_s to yield

$$Y = g_2\, A_s^{0.56} (\sin\beta)^{1.22} = g_3 T_c \tag{31}$$

Therefore, for the limiting case when $D_f >> D_i$, where D_i is the interrill soil loss per unit area, the LS factor therefore reduces to

$$LS = (A_s/22.13)^m (\sin\beta/0.0896)^n = T^*_c \tag{32}$$

where $m = 0.56$, $n = 1.22$, and T^*_c is Moore and Wilson's *(10)* dimensionless sediment transport capacity relationship.

Therefore, LS does indeed equal T^*_c and is a measure of the sediment transport capacity for this limiting case. This relationship indicates an upper limit to the slope length exponent, $m = 0.56$, and physical justification for the power function form for the slope factor S, as proposed by Moore and Wilson *(10)*. Based on comparisons with the observed LS data *(6)* this limiting case appears to occur for slope gradients > 12%.

Readers should note that there is a fundamental error in the development of the RUSLE slope-length relationship in McCool et al. *(6)*. They incorrectly use the following broad sheet flow equation in the derivation of the expression for effective shear stress [equation (7) in *(6)*].

$$\tau = \gamma y_b \sin\beta\, \xi^{2} \tag{33}$$

where γ is the weight density of the flow, y_b is the flow depth, and ξ is the ratio of flow velocity with cover and roughness to flow velocity over a bare, smooth soil. For rill flow, y_b should be replaced with the rill hydraulic radius, R_b. If this correct form of the hydraulic relationship is used, then an equation identical to equation (32) with m and n equal to 0.56 and 1.22, respectively, is obtained. This error, of using the incorrect form of the hydraulic relationship, propagates through their subsequent computations. The broad sheet flow equation produces equivalent values of m and n of 0.9 and 1.05. This explains why their upper limit on m is about 1.0 and why they might use a linear function of $\sin\beta$ for the slope factor, which is theoretically incorrect.

The other limiting case occurs when $D_f << D_i$, in which case interrill soil loss dominates and $m = 0$, as interill soil loss is independent of A_s or λ *(1)*. For this case, in very low slope gradient environments where no rills develop, soil loss for a catchment is likely to be controlled by the shallow sheet flow capacity to transport rainfall detached sediment (i.e., $q_s \geq T_c$), that is, by the transport capacity of shallow sheet flow. In such cases, the following equivalent LS factor can be derived:

$$LS = (A_s^{m2}/22.13^{m}) [(\sin\beta)^{n2}/0.0896^{n}] \tag{34}$$

where $m2 = -0.1$ (approximately = 0), $m = 0.56$, $n2 = 1.05$, and $n = 1.22$. For this equation, rilling is assumed to dominate for the reference condition where $A_s = 22.13$ m and $\tan\beta = 0.09$, and the exponents on these terms reflect a moderate rill/interrill ratio. This equation yields a slope factor that is almost a linear function of $\sin\beta$ and predicts a value of S of 0.073 for a 0.5% slope gradient, which is consistent with the empirical value from RUSLE. Therefore, the exponent on the slope length factor should range from 0 to 0.56, rather than from 0 to 1.0, using the RUSLE theory.

RUSLE slope steepness factor

The RUSLE S factors were purely empirically derived, which represents the second flaw in the development of the RUSLE S factors. McCool et al *(6)* developed the L factor using a theoretical approach proposed by Foster. This theoretical approach could easily have been applied to developing S factor relationships, but was ignored. Furthermore, McCool et al. *(5)* provide no physical justification for a distinct break in the S versus slope-gradient relationship at 9% within RUSLE. Moore and Wilson's *(10)* approach, which we have shown to be consistent with the detachment-limiting concept in RUSLE for the limiting case where rill erosion dominates, offers a theoretical basis for the development of the S factor relationships in RUSLE.

The slope steepness factor, S, as predicted by USLE, RUSLE, and the MW (Moore-Wilson) power functions, is compared in Table 1. For slope gradients >7%, RUSLE and MW predictions are within 0 to 5% of each other. For slope gradients of 3 to 12% (the range of slope gradients encompassing most of the data on which the USLE is based), the USLE and MW predictions are within 0 to 5%, whereas RUSLE predicts 31% larger values at a slope gradient of 4%. For slope gradients of 2%, RUSLE and MW predict S values that are 35% and −22% of the USLE values, respectively. At 1.5%, these values are 29% and −34%, respectively. For slope gradients ≤ 1%, MW predicts smaller values of S than either USLE or RUSLE.

RUSLE and USLE predict similar values of

S for slope gradients < 1%. The S function for low slope gradients in RUSLE is based on the results of Murphree and Mutchler *(11)*. The scatter of their data is large at small slope gradients and the fitted relationship used in RUSLE could have been forced through the origin (zero S - zero β), with little reduction in the coefficient of determination [see Figure 4 *(5)*].

Murphree and Mutchler *(11)* state that "soil loss on such low (0.1 to 3%) slopes was overestimated using the current (USLE) slope relationship." Furthermore, in discussing supporting evidence for the S relationship in RUSLE, McCool et al. *(5)* indicated that for 0.5% slopes the average S value was 0.13, with a standard deviation of 0.12 (ranging from 0 to 0.45). Hence, the MW predictions of S at low slope gradients fall within the one standard deviation error band. Also, the natural variation in slope gradient at low slopes makes application of the USLE and RUSLE relationships dubious. Only in special cases, such as in laser levelled fields, would it be appropriate to apply these relationships at low slope gradients.

However, as shown by equation (34), applying the transport-limiting concept at low slopes ($\leq$ 1%) produces an almost linear variation of S versus slope gradient and a value of S of 0.073 at a slope of 0.5%, which is similar to the RUSLE prediction. Hence, there is a theoretical justification for the low slope relationships in RUSLE.

The data supporting the break in slope of the S versus slope gradient relationship at a 9% slope gradient [Figures 1 and 2 *(5)*] are also weak, as many different curves could be fitted to the data with equal statistical validity. Hence, on purely empirical grounds, the MW S factor relationship is statistically as good as the RUSLE relationship. On intermediate slope gradients (3 to 9%), it appears to be superior, in that it more closely approximates the USLE estimates. Furthermore, the relationship has a physical basis from a consideration of the rill erosion-dominated and, hence, the transport-limiting condition.

Finally, for steep slopes in the Pacific Northwest wheat and range region, McCool et al. *(5)* have recommended a power function relationship for S, which has the same form as equation (32), but with n = 0.6 rather than 0.56. Hence, this recommended relationship is virtually identical to the MW relationship.

Slope length

Slope length is defined by Wischmeier and Smith *(14)* as "the distance from the point of origin of overland flow to the point where either the slope gradient decreases enough that deposition begins, or the runoff water enters a well defined channel that may be part of a drainage network or a constructed channel."

This definition is only valid for one-dimensional hillslopes and soil erosion research plots and under furrow irrigation and farming systems. The concept of slope length is quite inappropriate for three dimensional landscapes where the terrain causes convergence and divergence of overland flow. Such situations represent most applications of USLE and RUSLE. The reason that the slope length is inappropriate is that in USLE and RUSLE it is used as a surrogate of catchment area above a point, and hence of runoff and discharge. The concept of specific catchment area (drainage area per unit width orthogonal to the flow direction), as outlined by Moore and Wilson *(10)*, is conceptually and physically a far superior surrogate for runoff and discharge than is slope length. For a non-converging and non-diverging hillslope, the concept of specific catchment area is identical to that of slope length and so is compatible with existing approaches for this special case.

Renard et al. *(13)* say that "more questions and concerns are expressed about the L factor than any of the other USLE factors. One reason is that the choice of a slope length involves judgment, and different users choose different slope lengths for similar situations."

We certainly agree with this statement. One of the major reasons for this difficulty is that the concept of slope length is incorrect and inappropriate. However, it continues to be used and applied because of the inability of researchers and practitioners to escape from a one-dimensional view of the world. Research scientists argue about the values of the exponent in the length-slope factor in minute detail (e.g., Foster's comments), but seem happy to ignore the limitations of and the errors caused by the slope-length concept in applications to real landscapes. Another deficiency in the development of RUSLE has been the retention of the slope-length concept at a time when significant advances were being made in both terrain and spatial analysis, as illustrated below.

Soil loss in irregular terrain

Equation (20) in Moore and Wilson *(10)* hypothesized that the change in sediment transport capacity may possibly serve as a measure of the erosion and deposition potential in a cell. While this equation does represent the change in transport capacity for the transport-limiting case, what is really of interest is the soil loss per unit, Y, in a given cell. For the transport-limiting case, Foster has correctly shown that Y should be written as

$$Y_j = \phi \frac{[A_{sj}^{m+1} (\sin\beta_j)^n - A_{sj-}^{m+1} (\sin\beta_{j-})^n]}{\Delta x_j} \qquad (35)$$

where Y_j and Δx_j are the soil-loss and flow-path distance in cell ij, respectively, ϕ is a constant, and j and $j-$ signify the outlet and inlet of cell j, respectively.

We would like to build on Foster's discussion and show how the Foster and Wischmeier *(3)* approach for the detachment-limiting case can be expanded to more fully take into account terrain attributes such as slope, profile curvature, specific catchment area, and convergence and divergence of flow in complex 3-D terrain.

There is a large body of empirical evidence that suggests that the sediment flux, q_{sx} ($kg\ m^{-1}\ s^{-1}$) at any point in a catchment can be approximated by the following relationship:

$$q_{sx} = k_1\ q_x^{p+1} (\sin\beta)^n \qquad (36)$$

where q_x is the specific discharge and β is the slope angle at x, and p and n are exponents. The USLE and RUSLE soil loss equations are consistent with this relationship, if the reader is prepared to accept the arguments that follow, that the slope factor is well described by a power function relationship. In RUSLE and USLE, p varies with slope and is dependent on the susceptibility of the soil to rill erosion, relative to interrill erosion.

The specific discharge $q_x = A_{sx} r_x$, where A_{sx} is the specific catchment area ($m^2\ m^{-1}$) and r_x is the rainfall excess ($m^3\ m^{-2}\ s^{-1}$). The $\sin\beta$ term can be written as

$$\sin\beta = \frac{\partial z/\partial x}{\sqrt{1 + (\partial z/\partial x)^2}} \qquad (37)$$

where z is the elevation of the land surface and x is the distance in the aspect direction, i.e., in the direction of steepest slope. A useful approximation of equation (37) is $\sin\beta = \partial z/\partial x$, for which the error ranges from 0 at 0% slope to 4.4% at 30% slope. Therefore, equation (36) can be rewritten as

$$q_{sx} = k_1 (A_{sx} r_x)^{p+1} (\partial z/\partial x)^n \qquad (38)$$

The sediment loss per unit area, Y_x ($kg\ m^{-2}\ s^{-1}$), is $Y_x = \partial q_{sx}/\partial x$, yielding

$$Y_x = k_1 A_{sx}^p (\partial z/\partial x)^{n-1} [r_x^{p+1} [n A_{sx} (\partial^2 z/\partial x^2) + (p+1)(\partial z/\partial x)(\partial A_{sx}/\partial x)] + r_x^p (p+1) A_{sx} (\partial z/\partial x)(\partial r_x/\partial x)] \qquad (39)$$

If the rainfall excess per unit area is assumed uniform (i.e., $r_x = r$), as is commonly assumed in most simplified erosion models such as USLE and RUSLE, then $\partial r_x/\partial x = 0$ and equation (39) becomes

$$Y_x = k_1 A_{sx}^p (\partial z/\partial x)^{n-1} [n A_{sx} (\partial^2 z/\partial x^2) + (p+1)(\partial z/\partial x)(\partial A_{sx}/\partial x)]\ r^{p+1} \qquad (40)$$

This equation represents the effects of topography on soil loss in 3-D terrain for the detachment limiting case and is a more complete form of the equations derived by Foster and Wischmeier *(3)*. The terms in this equation A_{sx}, $\partial A_{sx}/\partial x$, $\partial z/\partial x$, and $\partial^2 z/\partial x^2$ are topographic attributes that can be readily calculated by terrain analysis *(8, 9)*.

For a non-converging or diverging hillslope (i.e., a simple hillslope), $\partial A_{sx}/\partial x = 1$ and equation (40) becomes

$$Y_x = k_1 A_{sx}^p (\partial z/\partial x)^{n-1} [n A_{sx} (\partial^2 z/\partial x^2) + (p+1)(\partial z/\partial x)] r^{p+1} \qquad (41)$$

This equation not only contains a slope term ($\partial z/\partial x$), but also a profile curvature term ($\partial^2 z/\partial x^2$) that describes the rate of change of slope. This last term is ignored in the Foster and Wischmeier *(3)* equation [Foster, equation (9)]. For a planar slope $(\partial^2 z/\partial x^2) = 0$ and equation (41) reduces to

$$Y_x = k_1 (p+1) A_{sx}^p (\partial z/\partial x)^n r^{p+1} = k_2 (p+1) A_{sx}^p (\sin\beta)^n \qquad (42)$$

which is the USLE and RUSLE equation for soil loss at a point.

Soil loss in irregular terrain: An exemplary catchment

We can illustrate the consequences of retaining an outmoded and inappropriate slope-length concept like that used in RUSLE by examining the *LS* values computed for a small agricultural catchment in Montana. The topographic map reproduced in Figure 1 was prepared from a plane table and geologic alidade survey by Pings *(12)*. The contours were digitized with ARC/INFO, exported to an ASCII file as a series of x,y,z points and converted to a regular grid with ANUDEM *(4)*. The TAPES-G *(7)* suite of terrain analysis programs was then used to compute the slope gradient, flowpath length and specific catchement area terms required for the RUSLE and MW LS equations [see Table 1, *(10)*] for each of the 689 30m × 30m cells. TAPES-G used digital versions of the catchment boundary and channel system mapped by Pings *(12)* to delineate the catchment boundary and channel system, respectively, the Rho8 (random-eight node) algorithm with a multiple drainage path option to determine flowpaths and upslope contributing areas for those cells above defined channels, and the Rho8 algorithm to direct flow below the cells marking points of

Table 1. Comparison of USLE, RUSLE and MW slope factors and USLE and RUSLE slope length exponents, m.

Slope (%)	S factors USLE	RUSLE	MW	L factor exponent, m USLE	RUSLE*
0.5	0.090	0.064	0.024	0.2	0.085
1.0	0.118	0.138	0.058	0.2	0.149
1.5	0.149	0.192	0.098	0.3	0.201
2.0	0.183	0.246	0.142	0.3	0.244
3.0	0.261	0.324	0.241	0.3	0.320
4.0	0.352	0.462	0.350	0.4	0.361
5.0	0.456	0.569	0.467	0.4	0.401
7.0	0.703	0.784	0.723	0.5	0.459
8.0	0.845	0.891	0.859	0.5	0.482
9.0	1.000	1.000	1.000	0.5	0.517
10.0	1.167	1.172	1.145	0.5	0.518
12.0	1.537	1.502	1.448	0.5	0.546
15.0	2.181	1.992	1.925	0.5	0.590
18.0	2.926	2.476	2.425	0.5	0.601
20.0	3.475	2.795	2.767	0.5	0.614
25.0	5.018	3.575	3.647	0.5	0.640
30.0	6.776	4.327	4.547	0.5	0.658

* For a moderate rill/interrill ratio

Table 2. Relative incidence of simple and complex hillslopes in a small Montana catchment.

Class	Ratio of A_s/λ	No. of cells	Slope gradient (%) Minimum	Mean	Maximum
1	≤ 0.40	1	3.35	3.35	3.35
2	0.41-0.80	15	0.21	5.14	19.44
3	0.81-1.20	103	0.21	6.61	23.14
4	1.21-1.60	123	0.54	7.08	33.91
5	1.61-2.00	110	0.00	5.22	30.06
6	2.01-3.00	144	0.00	5.98	27.49
7	3.01-5.00	128	0.26	6.96	25.82
8	5.01-10.0	62	0.12	5.42	19.95
9	>10.0	3	3.09	4.58	6.52
Totals		689	0.00	6.25	33.91

Table 3. Comparison of RUSLE and MW length-slope factors for selected cells in a small Montana catchment.

Cell ID	Slope (%)	λ (m)	A_s ($m^2 m^{-1}$)	Ratio of A_s/λ	RUSLE LS	MW LS
1	1.7	129.9	88.2	0.68	0.39	0.29
2	11.0	129.9	96.0	0.74	5.26	3.26
3	7.7	99.9	102.9	1.03	2.59	2.11
4	1.8	117.4	127.8	1.09	0.40	0.35
5	6.0	57.4	76.2	1.33	1.47	1.36
6	3.0	172.3	236.4	1.37	0.89	0.89
7	4.5	129.9	233.4	1.80	1.38	1.44
8	1.3	334.7	649.5	1.94	0.32	0.42
9	5.3	15.0	33.6	2.24	0.72	0.83
10	1.7	15.0	39.0	2.60	0.23	0.20
11	3.9	15.0	60.9	4.06	0.53	0.71
12	1.0	15.0	61.8	4.12	0.15	0.12

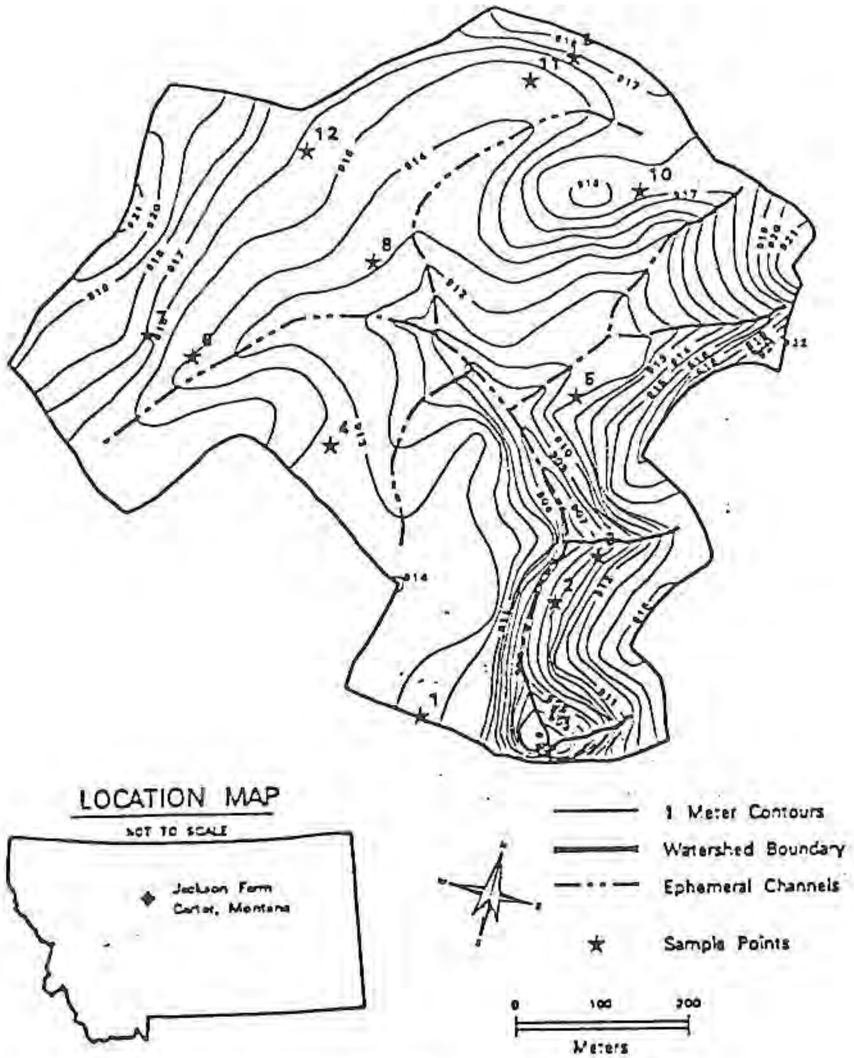

Figure 1. Map showing the 12 cells in the area surveyed.

channel initiation.

We can compare the flowpath length (m) and specific catchment area ($m^2 m^{-1}$) terms on a cell by cell basis to determine the relative incidence of planar hillslopes versus those where terrain causes convergence and divergence of overland flow. Table 2 shows the number of cells (and typical slope gradients) classified according to the ratio of specific catchment area to flowpath length. The third class contains only 103 cells and approximates the planar or one-dimensional hillslope for which the RUSLE slope-length concept is appropriate. Classes 1 and 2, and 4 through 7 contain the most cells (76%) and represent those cells located above the channel system on diverging and converging hillslopes, respectively. The final three classes with ratios of specific catchment area to flowpath length greater than 3.0 contain those cells that make up the 2.5 km long user-defined channel. We have reported minimum, mean, and maximum slope gradients with these data because the differences in the RUSLE and MW S estimates vary with slope gradient (Table 1) and these differences are carried forward with those due to converging and diverging hillslopes in Table 3 (discussed below).

Table 3 shows the flowpath lengths, specific catchment areas, slope gradients, RUSLE LS and MW LS values that were computed for the twelve cells located in Figure 1. Two cells, one representing gentle and the other steep slopes, were chosen from classes 2 through 7 of Table 2 for this part of the analysis. The LS equations reported in Table 1 of Moore and Wilson *(10)* were multiplied by an (m+1) term so that we could compute LS for individual cells in accordance with standard practice (as noted by Foster). These results capture the effects of using different equations for slope gradient (see Table 1 for details) and of including the effects of three-dimensional hillslopes in the MW equation. The cells with steep slopes minimize the differences due to slope gradient and the ordering of the cells from low to high ratios of specific catchment area to flowpath length meant that the effects of convergence of overland flow on the MW LS values becomes more pronounced from top to bottom in Table 3. Overall, the results in Tables 2 and 3 confirm that planar, one-dimensional slopes are very rare in nature and that the inclusion of terrain terms to account for the convergence and divergence of overland flow has a large impact on the computed LS values for the majority of cells.

Summary

The exponent m in RUSLE does vary and is bounded between 0 and 0.56, rather than 0 and 1 as proposed by McCool et al. *(5)* and Foster. This range also appears to fit the observed data very well *(6)*. In the limiting case where rill processes dominate, m = 0.56, and the *LS* factor is indeed a measure of the sediment transport capacity. A power function with exponent n = 1.22 (i.e., 1.3) at all but very low slopes (where the relationship is almost linear) fits the slope factor data as well as the existing RUSLE *S* factor relationship and has a physical basis. Equation (20) in Moore and Wilson *(10)* only represents the change in sediment transport capacity for the transport limiting case. Foster has demonstrated the appropriate equations for estimating soil loss per unit area. We have developed and demonstrated a more general form of the equations for the detachment limiting case that includes additional terrain attributes that enhances its application to 3-D terrain.

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Comments on Moore and Wilson

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Moore and Wilson presented an interesting development of a procedure to calculate LS factors for the RUSLE. The analysis, which appears to follow logically from their assumptions, is limited to rill erosion and the transport limiting case.

On the other hand, the developers of the LS relationships currently in use in RUSLE (*3,4*) relied heavily on field and plot erosion data including both rill and interrill erosion, with use of theoretical development to provide the forms of the relationships. The relationships did not assume, nor did the data indicate, transport limiting conditions.

A specific data set indicates limitations of the Moore and Wilson approach. Extensive rill erosion data were collected from fields in the Palouse Region of the Northwestern Wheat and Range Region (NWRR) from 1973 through 1983 (*2*). The data for complex slopes from 9 to 56% indicated slope length and steepness relationships, LS for erosion prediction of

$$LS = \left(\frac{\lambda}{22.13}\right)^{0.5} \left(\frac{\sin\theta}{\sin 5.14^{\circ}}\right)^{0.6}$$

where LS = slope length and steepness factors

λ = horizontal slope length in meters

θ = slope angle in degrees

No interrill erosion data were collected, and all measurements were taken above the footslope position where deposition occurred. In fact, we observed no deposition above the footslope.

When runoff and erosion events included rain or snowmelt on thawing soils we usually found larger amounts of deposition, extending slightly farther upslope, indicating that transport capacity was reached but only at the footslope where velocity decreased significantly. Erosion for even this thaw-weakened condition was apparently limited by detachment, not transport, for almost the entire slope length. The fitted slope steepness exponent of 0.6 is quite different from the theoretical value of 1.22 developed by Moore and Wilson. Use of the Moore and Wilson slope steepness exponent for the NWRR would result in overprediction of soil loss from steeper slopes of the region.

The assumption that detachment is directly related to the 1.5 power of tractive force t is an approximation to compensate for lack of knowledge of values of tc. Otherwise, detachment would be related directly to (t - tc). Soils in the NWRR are frequently thaw weakened at the time of erosion and tc should be quite small. This is reflected in the slope steepness exponent of 0.6. Following the assumptions and calculations of Moore and Wilson, and assuming detachment is directly related to tractive force t, then the slope length exponent for the NWRR data should be two-thirds of 0.56 or 0.37. This was not the case. It appears that the assumptions of Moore and Wilson are not truly general for erosion prediction from rills.

The authors indicate their procedure is appropriate for predicting soil loss from three-dimensional landscapes and watersheds. Such procedures would be useful because runoff from U.S. cropland almost always converges into a larger channel such as a concentrated flow channel or grassed waterway before exiting the field. However, it has not been proposed or assumed that RUSLE hydraulic processes would apply where concentrated flow erosion or deposition is a part of the runoff conveyance system. A major reason for the Water Erosion Prediction Project (WEPP) was the need for predicting erosion and deposition in concentrated flow channels. It is doubtful that the authors' LS relationship would make RUSLE more applicable to a strongly converging watershed.

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